Pre TPS

Dimensions and Units Tutorial
Solutions

1. a. MLT^{-2}; b. MLT^{-2}; c. L; d. T; e. L^3; f. ML^2T^{-2}

2. $M^{-1}L^3T^{-2}$ a. metric – $nt.m^2/kg^2$; b. imperial – $lbs.ft^2/slug^2$ (note that these are not exhaustive)

3. a. yes; b. no; c. yes; d. yes; e. no; f. yes.

4. 42,320 kms

5. ΔP = 52.76 lbs/sq.ft. = 0.37 lbs/sq.in. = 0.025 atmospheres

Name: _____

Algebra Tutorial Solutions

1. Combine:
   a. $2x + (3x - 4y)$ $5x - 4y$

   b. $4x^2 + 5x - (3x - 7) + (-2x^2 + 3)$ $2x^2 + 2x + 10$

   c. $[(x + 2y) - (x + 3y)] - [(2x + 3y) - (-4x + 5t)]$

   $-6x - 4y + 5t$

2. Add:
   a. $x^2 + 2x - 1 + 3x - 4 + 2x^2 + 5$ $3x^2 + 5x$

   b. $7x + 3y^3 - 4xy7, \quad x4 \quad y \quad xy$

8. Simplify:

a. $\dfrac{xy}{x \quad x}$

Name: _____

b.

Name: _____

Trigonometry Tutorial

1. Given:

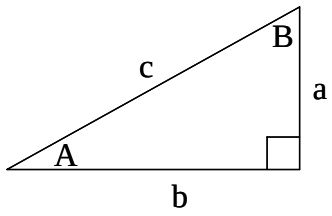
Find: sin A _____

cos A _____

tan A _____

2. Given: sin A = 2/5

c = 5



Find: a _____

b _____

$\angle B$ _____

3. $\sin 45 = \frac{1}{\sqrt{2}}$ $\sin 60 = \frac{\sqrt{3}}{2}$

cos 45 = _____ cos 60 = _____

tan 45 = _____ sin 30 = _____

sin 0 = _____ tan 60 = _____

cos 0 = _____

4. Given:

Find: sign of: $\tan\theta$ _____

$\sin\phi$ _____

$\cos\psi$ _____

$\sin\alpha$ _____

5. Given:

6. Show that:

$$\cos(\alpha+2\beta)=\cos\beta(\cos\alpha\cos\beta-\sin\alpha\sin\beta)-\sin\beta(\cos\alpha\sin\beta+\sin\alpha\cos\beta)$$

$$\begin{aligned}\cos(\alpha+2\beta)&=\cos\alpha\cos(2\beta)-\sin\alpha\sin(2\beta)=\cos\alpha\left(\cos^2\beta-\sin^2\beta\right)-2\sin\alpha\sin\beta\cos\beta=\\&=\cos\alpha\cos^2\beta-\sin\alpha\sin\beta\cos\beta-\cos\alpha\sin^2\beta-\sin\alpha\sin\beta\cos\beta=\\&=\cos\beta(\cos\alpha\cos\beta-\sin\alpha\sin\beta)-\sin\beta(\cos\alpha\sin\beta+\sin\alpha\cos\beta)\end{aligned}$$

7. Find the corresponding number of radians or degrees

a. 315 degrees — 7π/4

b. 120 degrees — 2π/3

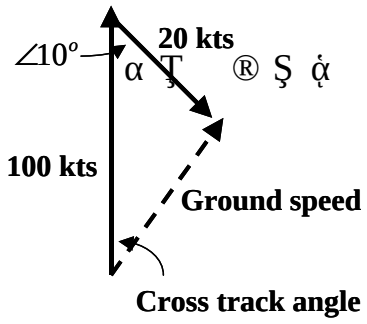
c. 100 degrees — 5π/9

d. π radians — 180°

e. $\frac{5\pi}{4}$ radians — 225°

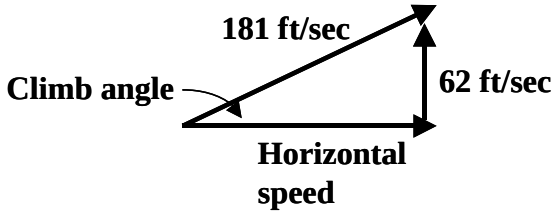
f. 1.6 radians — 91.72°

8. Given an aircraft traveling north at 100 kts into a 20 knot headwind from 350 degrees.



Find: Ground speed and cross track angle

9. Given an aircraft flying at 181 ft/sec and climbing at 62 ft/sec.

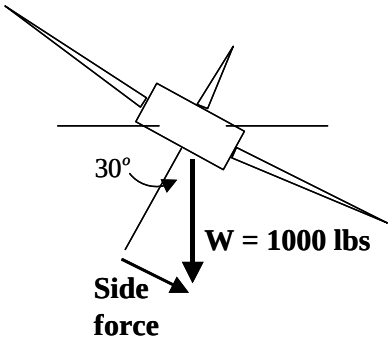


Find: Horizontal speed and climb angle.

$$HS = \sqrt{181^2 - 62^2} = 170\ ft/sec$$

$$CA = \sin^{-1}\ \frac{62}{181}\ = 20^\circ$$

10. Given a 1000 lb aircraft in a 30 degree bank.



Find: Side force.

Side Force = 1000 sin (30°) = 500 lbs

11. Given:

Assuming small angle theory (α is small), why is F_Z

Pre-TPS

Co-ordinate Systems and Graphs Solutions

1. a. +8; b. – 1; c. +3

2. L_1 is y = 2x; L_2 is y = x + 1; P_1 is (1, 2)

3.

a. Slope $P_2 – P_1$ = -1. Slope $P_1 – P_3$ = -1. Equation of line through P_1P_2 is y = -x + 1. This is satisfied by P3, (2, -1). Therefore all lie on same straight line.

b. Slope $P_1 – P_2$ = 2. Slope $P_2 – P_3$ = 2. Slope $P_3 – P_4$ = 2. Equation of line through P_2 with slope = 2 is y = 2x + 3. P_1, P_3 and P_4 all satisfy this. Therefore all are on same straight line.

4. Slope $P_1 – P_2$ = ¼. Slope $P_2 – P_3$ = -4. $m_1.m_2$ = -1. Therefore P_1P_2, P_2P_3 are perpendicular. Therefore $P_1P_2P_3$ is a right-angled triangle.

5. a. 3. b. -1. c. - ¾.

6. 2y = x + 3

7. $y = \sqrt{3}.x + (4 - \sqrt{3})$

Logarithms, Radicals and Exponents
Tutorial

1. Evaluate the following:

   a. $\left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 =$ $(1/2)^{3+2} = (1/2)^5$

   b. $\frac{a^{10}}{a^4} =$ $a^{10-4} = a^6$

   c. (

3. Write the following in logarithmic form:

a. $7^2 = 49$ $\quad$ $\text{Log}_7 49 = 2$

b. $3^2 = 27$ $\quad$ $\text{Log}_3 27 = 3$

c. $2^{-3} = \frac{1}{8}$ $\quad$ $\text{Log}_2 (1/8) = -3$

d. $\sqrt[3]{8} = 2$ $\quad$ $\text{Log}_8 2 = 1/3$

4. Write the following in exponential form:

a. $\log_3 81 = 4$ $\quad$ $3^4 = 81$

b. $\log_9 27 = \frac{3}{2}$ $\quad$ $9^{3/2}$

6. Find:

a. $\log_{10} 3860$	3.5866
b. $\log_{10} 5.46$	0.7372
c. $\log_{10} .00235$	-2.6289
d. $\log_{10} .0000129$	-4.8894
e. $\log_{10} 72800$	4.8621

7. Solve for x:

a. $(3)(10^{x}) = 27$	0.9542
b. $e^{2(x-5)} = 30$	6.7
c. $2e^{x} = 8$	1.3863
d. ln x – ln (x-1) = 2	1.156

Complex Numbers Tutorial

Solutions

1. Perform the indicated operations:

   a. $(3 - 4i) - (-5 + 7i)$ — $8 - 11i$

   b. $(4 + 2i) + (-1 + 3i)$ — $3 + 5i$

   c. $(2 + i)(3 + 2i)$ — $4 + 7i$

   d. $(3 - 4i)(3 + 4i)$ — 25

   e. $\frac{1+3i}{2-i}$ — $-1/5 + (7/5)i$

   f. $\frac{3-2i}{2+3i}$ — $-i$

2. Find the conjugate of the following:

   a. $2 + i$ — $2 - i$

   b. $2 - 3i$ — $2 + 3i$

   c. $-4 + 2i$ — $-4 - 2i$

   d. $-4 - 3i$ — $-4 + 3i$

   e. $3i - 7$ — $-7 - 3i$

3. Graph the following:

    a.

5. Express the following in polar form:

a. $+1+i\sqrt{3}$ _____

b. $6\sqrt{3}+6i$ _____

c. $0 + 4i$ _____

d. $-1 + i$ _____

6.

7. Use De Moivre's theorem to evaluate the following and express results in a + bi form:

a. $\left(1+\sqrt{3}i\right)^5$ $\underline{\quad 16-16\sqrt{3}i \quad}$

$$\left(1+\sqrt{3}i\right)^5 = 2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^5 = 32\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right) =$$

$$= 32\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right) = 16-16\sqrt{3}i$$

b. $\sqrt{\left(1-\sqrt{3}i\right)}$ $\underline{\quad \sqrt{6}-\sqrt{2}i \quad}$

$$\sqrt{\left(1-\sqrt{3}i\right)} = \left(1-\sqrt{3}i\right)^{1/2} = 2\left(\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right)^{1/2} =$$

$$= \sqrt{2}\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right) =$$

$$= \sqrt{2}\left(\frac{\sqrt{3}}{2}-i\frac{1}{2}\right) = \frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}i$$

8. Express the following in the alternate forms requested:

a. 4(cos 60° + i sin 60°) exponential form: $4e^{i\pi/3}$

b. $6\sqrt{3}+6i$ $\sqrt{}$ exponential form: $12e^{i\pi/6}$

e. $4e^{\frac{\pi}{2}i}$ polar form: 4 (cos90° + i sin90°)

rectangular form: 0 + 4i

Determinant & Matrix
Tutorial

1. Solve the following determinants:

a. $\begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix}$ -2

b. $\begin{vmatrix} -3 & -4 \\ 2 & 7 \end{vmatrix}$ -13

c. $\begin{vmatrix} 1 & 1 & 1 \\ 3 & -3 & -3 \\ 1 & -1 & 2 \end{vmatrix}$ -18

d. $\begin{vmatrix} 3 & -1 & 1 \\ 5 & 6 & 4 \\ 0 & 1 & 2 \end{vmatrix}$ 39

2. Solve the following using Cramer's Rule

$x + y + z = 0$	$x = 2$
$3x – 3y - 3z = 12$	$y = 5/3$
$x –y + 2z = -7$	$z = -11/3$

3. Add or subtract the following matrices

a. $\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} -3 & -4 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 5 & 12 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 1 & 1 \\ 3 & -3 & -3 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -1 & 1 \\ 5 & 6 & 4 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 2 & 0 \\ -2 & -9 & -7 \\ 1 & -2 & 0 \end{bmatrix}$

Vector Algebra Tutorial
Solutions

1. Length of vector $= \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2}$

$= \sqrt{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 1$

2. $\sqrt{a^2+b^2+c^2} = \sqrt{4+9+1} = \sqrt{14}$

unit vector is $\frac{2}{\sqrt{14}}\hat{i} + \frac{3}{\sqrt{14}}\hat{j} - \frac{1}{\sqrt{14}}\hat{k}$

3. Length of $\overline{A} = \sqrt{14}$

Length of $\overline{B} = \sqrt{56}$

4. $\left|\overline{\quad 2\hat{\ }\ \hat{\ }\ }\right| = \sqrt{5}$

Pre-TPS
Differentiation
Solutions

1. *a.* $f(x) = 2x^2 + x$ — $4x + 1$

   b. $f(x) = 3x^2 + 2x + 1$ — $6x + 2$

2. a. $\frac{d}{dx}\left(2x^2 + x\right)$ — $4x + 1$

   b. $\frac{d}{dx}\left(3x^3 - 4x^2 + 5x - 2\right)$ — $9x^2 - 8x + 5$

   c. $\frac{d}{dx}\left(2u^2 v\right)$ where u and v are functions of x — $2u^2(dv/dx) + 4uv(du/dx)$

   d. $\frac{d}{dx}\left(1/2x^4 + 5x\right)$ — $2x^3 + 5$

   e. $\frac{d}{dx}\left(2u^2 / v^3\right)$ where u and v are functions of x — $\frac{4uv\frac{du}{dx} - 6u^2\frac{dv}{dx}}{v^4}$

3. $y = 2z^2 + z$ and $z = (x - 2)$; chain rule gives $\frac{dy}{dx} = (4z + 1).1$

4. $y = x^3 + 4x + 3$; $\frac{dy}{dx} = 3x^2 + 4$

5. $y = \frac{x^2 - 3}{x + 4}$; $\frac{dy}{dx} = \frac{x^2 + 8x + 3}{(x + 4)^2}$

6. $y^2 + x - 4 = 0$; $\frac{dy}{dx} = -1/2y$

7. $x^2 + 2xy - 3y^2$

c. $y = (x + 2)(x - 3)$ 2

d. $y - x^2 - 12 = x^7 + 3x^4 + 4x^2 - x + 10$ $42x^5 + 36x^2 + 10$

9. $s = 120t - 16t^2$

velocity, $ds/dt = 120 - 32t$; acceleration, $d^2s/dt^2 = -32$

velocity at t = 2 = 56 ; acceleration at t = 2 = -32

10. maximum and minimum values for x and y below are:

a. $y = x^3 + 2x^2 - 15x - 20$ x = -3 or +5/3; y = +16 or -34.81

b. $y = x^2 - 10$ x = 0; y = -10

PreTPS
Integration
Solutions

1. Integrate the following non-definite integrals

a. $\int\left(x^3+6x^2+7\right)dx$ $x^4/4 + 2x^3 +7x + C$

b. $\int\frac{dx}{x^2}$ $-1/x + C$

c. $\int\frac{2x+1}{\left(x^2+x\right)}dx$ $Ln(x^2 + x) + C$

d. $\int\sin 3x\,dx$ $\frac{-1}{3}\cos 3x + C$

e. $\int\left(\cos 4x+\sec^2 x\right)dx$ $\frac{1}{4}\sin 4x+\tan x+C$

f. $\int e^{3x}\,dx$ $\frac{1}{3}e^{3x}+C$

2. Evaluate the following definite integrals:

a. $\int_0^{\frac{\pi}{2}} 3\sin x\,dx$ +3

b. $\int_{-\pi}^{+\pi} 2\cos x\,dx$ 0

c. $\int_0^3\left(x^2+7x+6\right)dx$ 117/2

3. Integrate by parts:

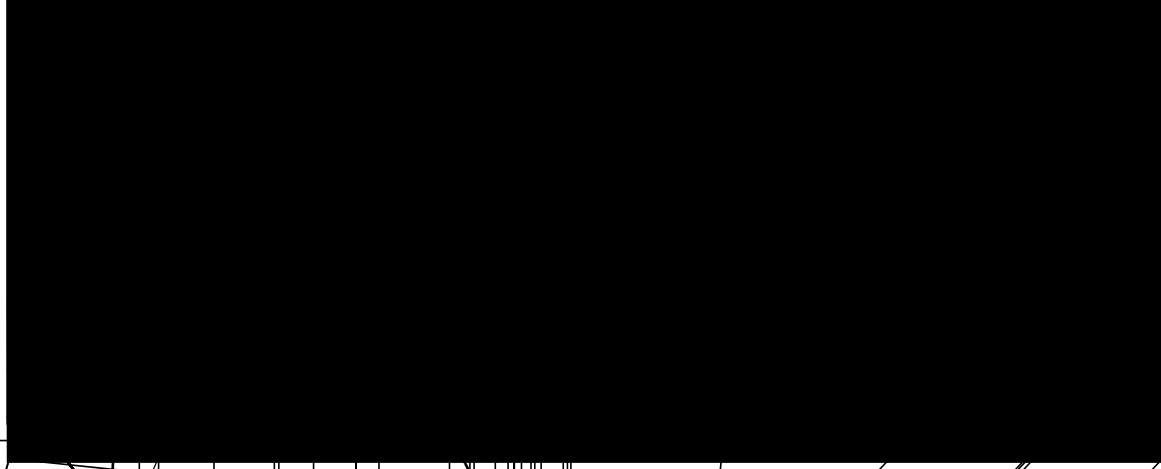
$\int x\cdot\sin x\,dx$ Sinx – xCosx + C

4. Integrate by substitution:

$\int\sin^3 x\cos x\,dx$ ¼ $Sin^4x + C$

5. Find the area under the curve $y = x^3 + 3x^2 + 2$ between $x = 0$ and $x = 2$. 16 sq units

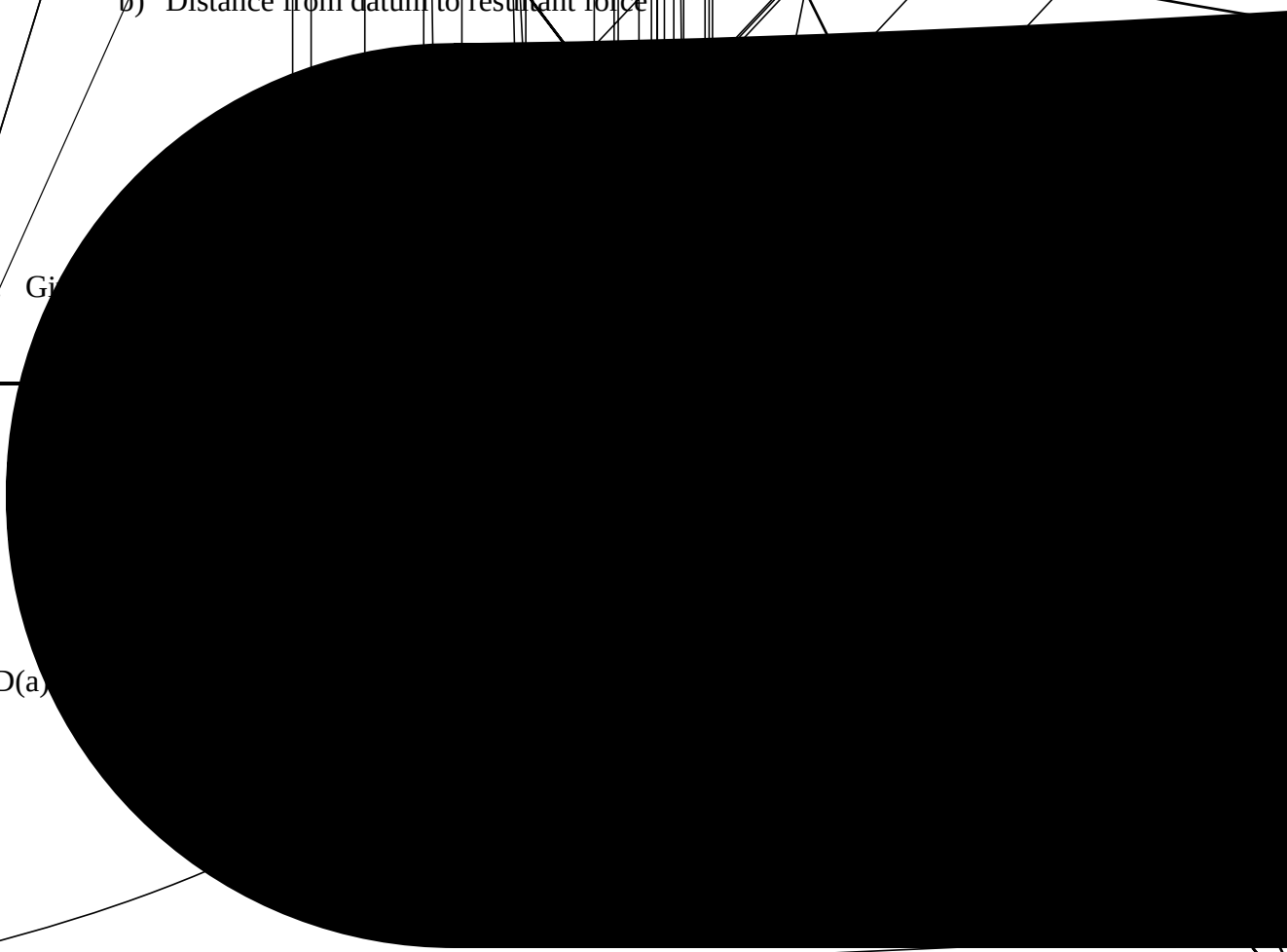
3. Given:



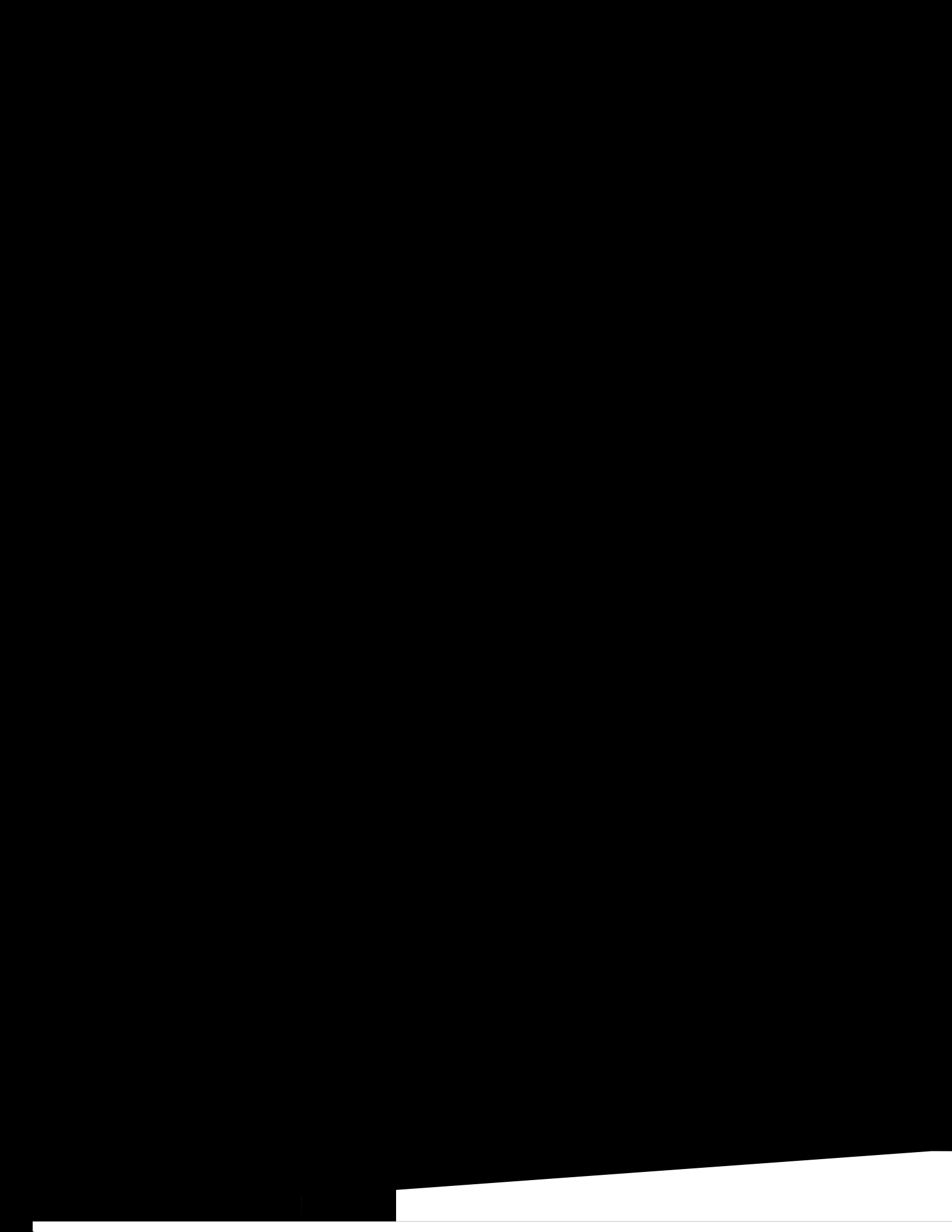
Find: a) Resultant force

b) Distance from datum to resultant force

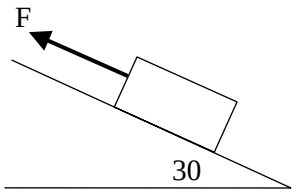
4. Gi



ep.8D(a)



7. Given:



Weight of block = 150 lbs

$\mu_s = 0.3$

Find: a) Minimum force required to hold the block at rest.

$$N = W\cos 30^\circ = 150\frac{\sqrt{3}}{2} = 130lbs$$

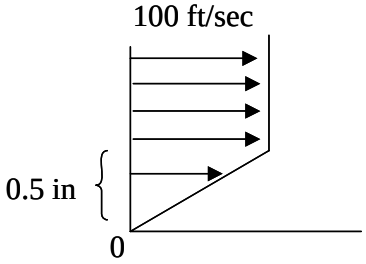
$$f_{max} = \mu_s N = 39lbs$$

$$F_{min} = W\sin 30^\circ - f_{max} = 75 - 39 = 36lbs$$

b) Maximum force required to hold the block at rest.

$$F_{max} = W\sin 30^\circ + f_{max} = 75 + 39 = 114lbs$$

8. Given:



Calculate $\frac{dV}{dy}$ from this data

$$\mu = 1.2x10^{-5}\frac{lb-\sec}{ft^2}$$

Find: a) $\frac{dV}{dy}$

$$\frac{dV}{dy} = \frac{100\ ft/sec}{0.5in} = \frac{100\ ft/sec}{0.5\ ft/12} = 2{,}400/sec$$

b) τ

$$\tau = \mu\frac{dV}{dy} = 28.8\cdot 10^{-3}\frac{lb}{ft^2}$$

c) Shear force acting over a 200 ft^2 area

$$F = \tau\cdot S = 28.8\cdot 10^{-3}\cdot 200 = 5.76lb$$

9. Consider an aircraft weighing 125,000 lbs taxing on the ground.

Assuming that:

- the reaction force on the nose wheel is 25,000 lbs;
- the reaction force on the main gear is 100,000 lbs (50,000 lbs per wheel)
- the radius of the nose wheel is 25 in;
- the radius of the main gear wheel is 50 in;
- the coefficient of rolling resistance is b=1 in;
- the aerodynamic drag is negligible;

find the engine thrust necessary to maintain a constant ground speed

10. Assuming μ_s=0.4, calculate the maximum braking force the crew can apply without skidding.

Work and Energy Tutorial

1. Determine the amount of work performed on the block when it is moved 10 ft UP the incline as shown. ASSUME NO FRICTION

2.

4. A 10,000 lb fighter experiences an engine flame-out at 30,000 ft and 1200 ft/sec airspeed. Assuming no energy losses during a zoom climb, calculate the maximum altitude when the aircraft reaches 500 ft/sec velocity.

$$SE = H + \frac{1}{2}\frac{V^2}{g} = 30{,}000 + \frac{1}{2}\frac{1{,}200^2}{32.2} = h_{max} + \frac{1}{2}\frac{500^2}{32.2}$$

$$h_{max} = 30{,}000 + \frac{1}{64.4}\left(1{,}200^2 - 500^2\right) = 48{,}478\ ft$$

5. An aircraft is flying at 35,000 ft and 1000 ft/sec airspeed. The aircraft weighs 35,000 lbs.
   a. Find the specific energy of the aircraft

$$SE = H + \frac{1}{2}\frac{V^2}{g} = 35{,}000 + \frac{1}{2}\frac{1{,}000^2}{32.2} = 50{,}528\ ft$$

   b. Assuming no losses, find the maximum velocity of the aircraft at sea level.

$$SE = \frac{1}{2}\frac{V^2}{g} = 50{,}528\ ft$$

$$V = \sqrt{2 \cdot 32.2 \cdot 50{,}528} = 1804\ ft/sec$$

6. A spring is compressed 6 inches. (K = 300 lb/in) If a 50 lb object is placed on top of the compressed spring and the spring is released.
   a. What is the spring force before release?

$$F = K \cdot x = 300 \cdot 6 = 1800lb$$

   b. Calculate the stored energy in the spring before it is released.

$$E = \frac{1}{2}Kx^2 = \frac{1}{2} \cdot 300 \cdot 36 = 5400in \cdot lb = 450\ ft \cdot lb$$

   c. What is the velocity of the object at separation from the spring. (assume Stored energy = KE)

$$\frac{1}{2}\frac{W}{g}V^2 = \frac{1}{2}Kx^2 = 450\ ft \cdot lb$$

$$V = \sqrt{\frac{2 \cdot g \cdot 450}{W}} = \sqrt{\frac{2 \cdot 32.2 \cdot 450}{50}} = 24\ ft/sec$$

[redacted]

aws

olutions

[redacted]

n steady, unaccelerated flight? Why?

No. Net horizontal forward force 500 lbs

the aircraft acceleration

$5.33 ft/sec^2$

T = D = 2000 lbs

a. Is the aircraft in level flight? Why?

No. Vertical forces not balanced.

b. Calculate the vertical acceleration

$9.6 ft/sec^2$ descent

L = 3000 lbsW = 3000 lbsT = 2000lbsD =1500bs

3. Compute RAF

RAF = 50 lbs

4.

a. In order to hold the elevator in place how much hinge moment (H) is required?

+25 ft-lbs

b. If the elevator hinge suddenly breaks, what will be the horizontal acceleration of the elevator surface [Assume the elevator weights 10 lbs]

227.6 ft/sec^2

Inertia Tutorial

1.

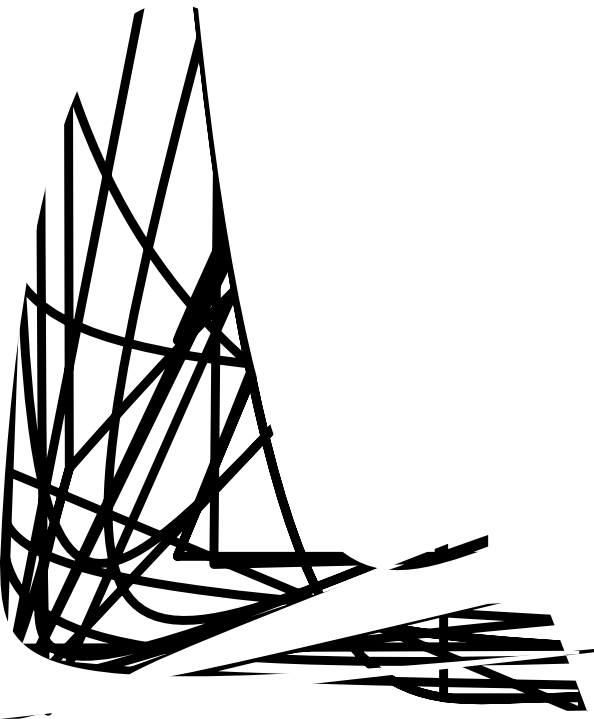
a. Find I_y for the aircraft loaded as shown above (empty weight moment of inertia: $(I_y)_{empty}$=80,000 slug·ft^2).

b. Find I_y of the aircraft after the 1000 lb payload is dropped from the forward bay.

3. Given the following future X-airplane, calculate the mo
the products of inertia (Ixy, Iyz, Ixz).

Momentum and Impulse
Tutorial Solutions

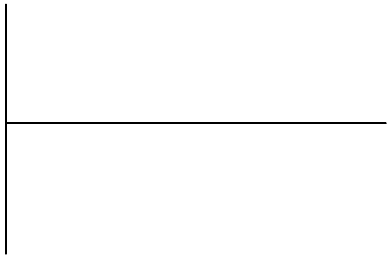
1. An 8gm bullet is fired horizontally into a 9kg block of wood which is free to move. The velocity of the block and bullet after impact is 40cm/sec. Calculate the initial velocity of the bullet. (V = 45,040cm/sec)



Motion Analysis
Tutorial Solutions

1. Draw a typical trace for the following oscillating system.
   a. Positive damped (stable)

   b. Neutral damped (neutral)



   c. Negative damped (unstable)

2. Given the following 1st order response

   a. Estimate τ = 1 sec

   b. Write the time history response equation x(t) = 100(1-e^{-t})

   c. Is the response convergent or divergent? convergent

3. Given the following "s-domain" equations

   (1) s + .0095 = 0
   (2) $s^2 + .875s + 18.4 = 0$

   a. Find time constant (τ) $s + \frac{1}{\tau} \Rightarrow 0.0095 = \frac{1}{\tau} \rightarrow \tau = 105\text{sec}$

   b. Find natural frequency (ω_n) $\omega^2 = 18.4 \rightarrow \omega = 4.29$

   c. Find damping ratio (ξ) .10

4. Given the attached trace, calculate the damping ratio (ξ) using the Transient Peak Method.

Average TPR = .59
Thus damping ration = 0.17

5. Given the following, calculate the time constant (τ) using $\tau = \dfrac{\Delta t}{\ln \dfrac{A_1}{A_2}}$

$$\tau = \frac{1}{\ln \frac{30}{20}} = 2.4$$